Topic: Linear approximation to estimate a root

Question: What is the closest linear approximation of $\sqrt[3]{9}$?

Answer choices:

A $\frac{41}{12}$

B $\frac{23}{12}$

C $\frac{7}{12}$

D $\frac{25}{12}$

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Solution: D

To solve, we need to find a similar cube root for which we know the solution. We can use $2^3 = 8$. Then we can turn our given cube root into the function

$$f(x) = \sqrt[3]{x}$$

For our approximation, we can say that $a = 8$, and then we can find find $f(a)$.

$$f(a) = \sqrt[3]{a} = a^{\frac{1}{3}}$$

And $f(8)$ is

$$f(8) = \sqrt[3]{8}$$

$$f(8) = 2$$

Then $f'(a)$ is

$$f'(a) = \frac{1}{3}a^{-\frac{2}{3}}$$

And so $f'(8)$ is

$$f'(8) = \frac{1}{3}(8)^{-\frac{2}{3}}$$

$$f'(8) = \frac{1}{3}(64)^{-\frac{1}{3}}$$

$$f'(8) = \frac{1}{3}(4)^{-1}$$

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$$f'(8) = \frac{1}{12}$$

Now we can create our linear approximation model using $f(8) = 2$, $f'(8) = 1/12$, and $a = 8$.

$$L = f(a) + f'(a)(x - a)$$

$$L = 2 + \frac{1}{12}(x - 8)$$

$$L = 2 + \frac{x}{12} - \frac{8}{12}$$

$$L = \frac{24}{12} + \frac{x}{12} - \frac{8}{12}$$

$$L = \frac{16}{12} + \frac{x}{12}$$

$$L = \frac{16 + x}{12}$$

Finally we can take the original root $\sqrt[3]{9}$ where $x = 9$ in relation to our function $f(x) = \sqrt[3]{x}$ and the value we found for L to find the linear approximation.

$$L = \frac{16 + 9}{12}$$

$$L = \frac{25}{12}$$

The linear approximation of $\sqrt[3]{9}$ is $25/12$.

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Topic: Linear approximation to estimate a root

Question: What is the closest linear approximation of $\sqrt[3]{29}$?

Answer choices:

A $\frac{83}{27}$

B $\frac{25}{27}$

C $\frac{137}{27}$

D $\frac{79}{27}$

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Solution: A

To solve, we need to find a similar cube root for which we know the solution. We can use $3^3 = 27$. Then we can turn our given cube root into the function

$$f(x) = \sqrt[3]{x}$$

For our approximation, we can say that $a = 27$, and then we can find find $f(a)$.

$$f(a) = \sqrt[3]{a} = a^{\frac{1}{3}}$$

And $f(27)$ is

$$f(27) = \sqrt[3]{27}$$

$$f(27) = 3$$

Then $f'(a)$ is

$$f'(a) = \frac{1}{3}a^{-\frac{2}{3}}$$

And so $f'(27)$ is

$$f'(27) = \frac{1}{3}(27)^{-\frac{2}{3}}$$

$$f'(27) = \frac{1}{3}(729)^{-\frac{1}{3}}$$

$$f'(27) = \frac{1}{3}(9)^{-1}$$

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$$f'(27) = \frac{1}{27}$$

Now we can create our linear approximation model using $f(27) = 3$, $f'(27) = 1/27$, and $a = 27$.

$$L = f(a) + f'(a)(x - a)$$

$$L = 3 + \frac{1}{27}(x - 27)$$

$$L = 3 + \frac{x}{27} - \frac{27}{27}$$

$$L = 3 + \frac{x}{27} - 1$$

$$L = 2 + \frac{x}{27}$$

$$L = \frac{54}{27} + \frac{x}{27}$$

$$L = \frac{54 + x}{27}$$

Finally we can take the original root $\sqrt[3]{29}$ where $x = 29$ in relation to our function $f(x) = \sqrt[3]{x}$ and the value we found for L to find the linear approximation.

$$L = \frac{54 + 29}{27}$$

$$L = \frac{83}{27}$$

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The linear approximation of $\sqrt[3]{29}$ is $83/27$.

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Topic: Linear approximation to estimate a root

Question: What is the closest linear approximation of $\sqrt[4]{79}$?

Answer choices:

A $\frac{322}{54}$

B $\frac{164}{54}$

C $\frac{161}{54}$

D $\frac{82}{54}$

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Solution: C

To solve, we need to find a similar cube root for which we know the solution. We can use $3^4 = 81$. Then we can turn our given cube root into the function

$$f(x) = \sqrt[4]{x}$$

For our approximation, we can say that $a = 81$, and then we can find find $f(a)$.

$$f(a) = \sqrt[4]{a} = a^{\frac{1}{4}}$$

And $f(81)$ is

$$f(81) = \sqrt[4]{81}$$

$$f(81) = 3$$

Then $f'(a)$ is

$$f'(a) = \frac{1}{4}a^{-\frac{3}{4}}$$

And so $f'(81)$ is

$$f'(81) = \frac{1}{4}(81)^{-\frac{3}{4}}$$

$$f'(81) = \frac{1}{4}(531{,}441)^{-\frac{1}{4}}$$

$$f'(81) = \frac{1}{4}(27)^{-1}$$

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$$f'(81) = \frac{1}{108}$$

Now we can create our linear approximation model using $f(81) = 3$, $f'(81) = 1/108$, and $a = 81$.

$$L = f(a) + f'(a)(x - a)$$

$$L = 3 + \frac{1}{108}(x - 81)$$

$$L = 3 + \frac{x}{108} - \frac{81}{108}$$

$$L = \frac{324}{108} + \frac{x}{108} - \frac{81}{108}$$

$$L = \frac{324}{108} + \frac{x}{108} - \frac{81}{108}$$

$$L = \frac{243}{108} + \frac{x}{108}$$

$$L = \frac{243 + x}{108}$$

Finally we can take the original root $\sqrt[4]{79}$ where $x = 79$ in relation to our function $f(x) = \sqrt[4]{x}$ and the value we found for L to find the linear approximation.

$$L = \frac{243 + 79}{108}$$

$$L = \frac{322}{108}$$

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$$L = \frac{161}{54}$$

The linear approximation of $\sqrt[4]{79}$ is $161/54$.

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